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# The mixed state of a superlattice of superconducting sheets

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**Abstract.** We have investigated the mixed state of a superlattice of superconducting sheets, which is a model for the cuprate superconductor and the superlattices made from them. We have also studied the relevant length scales. It is shown that only when the separation between the sheets $\ell$ is larger than $\lambda_{eff} = \lambda^2/d$ can the system be viewed as a collection of separate sheets; here $\lambda_{eff}$ is the screening length for a single sheet. It is argued that there are *two* Abrikosov lattices; one is associated with the component of the field which is parallel to the sheets, while the second corresponds to the field component perpendicular to the sheets. It is shown that the two lattices interact and can form a commensurate structure with novel properties similar to those found experimentally.

## 1. Introduction

The very anisotropic properties of the the high-temperature cuprate superconductors have led many to believe that, in essence, they consist of a two-dimensional superlattice of superconducting sheets which are weakly Josephson coupled.

In this paper the mixed state of such a 'superstack' of weakly coupled thin films will be investigated; this is a model for the cuprate superconductors themselves (see,for example, Dolan *et al* 1989 and Bolle *et al* 1991) *and* the superlattices (see Brunner *et al* 1991 and Norton *et al* 1991) fabricated from them. The mixed state is defined as the state of a type II superconductor in which there exists one or more Abrikosov flux lattices.

The calculations reported here serve to illustrate the role of various length scales. With a sufficiently large separation $\ell$ between the sheets (see figure 1) the system must behave as a set of isolated sheets, while for a small enough separation the composite should behave like a bulk type II superconductor; this is for the situation when the field $\boldsymbol{B}$ is perpendicular to the sheets. Given a London penetration depth, $\lambda$ (intrinsic to the material of the sheets), and $d$, the thickness of the sheets, then the possible lengths are $\lambda' = (\ell/d)^{1/2}\lambda$ (a modified London penetration length) and $\lambda_{eff} = \lambda^2/d$ (the screening length for a single sheet along with $\ell$ itself). Only when $\ell > \lambda_{eff}$ does the system behave like a collection of single sheets. When this inequality is not satisfied $\lambda'$ plays the role of the bulk London length.

For the case when $\boldsymbol{B}$ is *not* perpendicular to the sheets it is widely assumed (see Ivlev and Kopnin 1991, and references therein) that, given $\ell \ll \lambda_{eff}$, the mixed state of such a superstack is equivalent to the similar state of a highly anisotropic *uniform*

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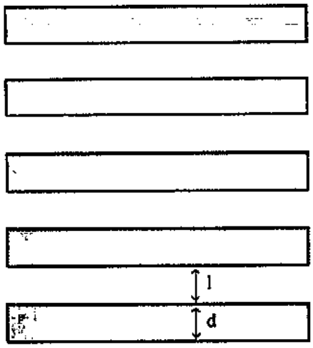


**Figure 1.** The stack consists of superconducting sheets of thickness $d$ with a space $\ell$ between them. In the formal calculations, the superconducting sheets are treated as if the sheets have no thickness, i.e. the limit $d \to 0$ is taken whilst maintaining fixed current density, etc.

superconductor (i.e. usually theories are constructed using the Landau–Ginzberg theory with a highly anisotropic mass tensor). Here, in section 2, it is argued that this description is not even qualitatively correct. Rather, if the Josephson coupling is not too strong, there are *two* Abrikosov lattices; one, the *parallel lattice*, is associated with the component of the field which is parallel to the sheets of the stack while the second, the *perpendicular lattice*, corresponds to the component perpendicular to the sheets.

The model, introduced in section 3, *is* based upon the Landau–Ginzberg equations. However, the superconductivity is confined to a set of thin sheets, i.e. it is highly non-uniform.

The calculations are presented in section 4, 5 and 6. The Landau–Ginzberg equations for the superstack are solved in section 4 and the perpendicular and parallel values of $B_{c1}$ are calculated in section 5 and 6. The introduction, in section 6, of the Josephson coupling between the sheets brings another length $\lambda_J$ (the Josephson depth) into the picture. The Josephson depth and $\lambda'$, the effective London penetration depth, are related to the lower critical fields $B_{c1}^{\perp}$ and $B_{c1}^{\parallel}$, in an obvious notation.

It will be shown in section 7 that, because the normal cores of the perpendicular lattice suppress the local Josephson current, there is a coupling energy which correlates the perpendicular and parallel lattices. For sufficiently strong coupling the two lattices will form a commensurate structure. When this is the case then the coupling will (i) orientate the perpendicular lattice and (ii) possibly form chains; the latter effect in order to maintain commensurability between the two lattices. Such effects have been observed by Bolle *et al* (1991). Section 8 contains the conclusions.

## 2. One versus two Abrikosov lattices

As stated in the introduction, it is widely assumed (see Ivlev and Kopnin 1991, and references therein) that the mixed state of the materials of interest corresponds to a highly anisotropic *but uniform* superconductor—usually theories are constructed using Landau–Ginzberg theory with a highly anisotropic mass tensor; such a theory implies the existence of a *single* Abrikosov lattice. By contrast, in the superstack model Landau–Ginzberg theory is still assumed *but* the superconductivity is localized in sheets (i.e. it is highly non-uniform); it will be argued that such a model predicts two Abrikosov lattices.

The energy per unit length for a homogeneous anisotropic superconductor depends upon the angle between the field and the planes; $\sim \phi_0^2/\lambda'^2$ when they are perpendicular (where $\lambda'$ is the effective London penetration depth for the field perpendicular to the sheets and $\phi_0$ is the flux quantum) and $\sim \phi_0^2/\lambda'\lambda_J$ when they are parallel (where $\lambda_J$ is the Josephson penetration depth). For intermediate angles Ivlev and Kopnin (1991), for example, suggest that the appropriate dependence is an energy which varies as $\sim \lambda(\theta)\phi_0^2/\lambda'^2\lambda_J$, where $\lambda^2(\theta) = \lambda'^2 \sin^2\theta + \lambda_J^2 \cos^2\theta$ and $\theta$ is the angle between the field and the $z$ axis. If instead the superconductivity is localized in very thin sheets then one is led to the idea of 'pancake' vortices strictly localized in the sheets and, as a first estimate, the energy cost of a vortex is simply given by the number of sheets it crosses, i.e. $\sim \phi_0^2/\lambda_{\rm eff}$ per sheet.

In the limiting case $\lambda_J \to \infty$ the coupling between pancake vortices lying in different sheets is purely electromagnetic. This problem has been considered previously by Clem (1991). In the calculation presented in section 4, the problem of a superstack is solved by considering a single sheet which is repeated periodically using Fourier series for the $z$ dependence of the various physical quantities. In order to consider the problem of the rigidity of a vortex, it is necessary to at least consider two sheets which are then repeated by the Fourier series techniques. Although the details are not given here, this problem has also been solved by the author. It is found that the energy cost for a vortex tilted by an an angle $\sim 45°$ is $\sim (\phi_0^2/\lambda_{\rm eff})(\ell/\lambda_{\rm eff})$ per vortex line per sheet; this differs from the result of Clem (1991). Whatever the merits of the two calculations, it is clear that there is a finite energy required to tilt the Abrikosov lattice and so, for sufficiently weak Josephson coupling when the field makes an appreciable angle to the $z$ direction, it is always an advantage to establish two rather (than a single) Abrikosov lattices.

In the presence of a Josephson coupling, the energy cost per unit length of a vortex parallel to the sheets is $\sim \phi_0^2/\lambda'\lambda_J$, and the comparable energy per sheet is $\sim \phi_0^2\ell/\lambda'\lambda_J$. The various lengths have fairly large uncertainties. For $YBa_2Cu_3O_7$, it might be reasonable to take $\lambda \sim 10^3$ Å, $\lambda_J \sim 10^4 - 10^5$ Å and $d \sim 10$ Å, whence $\lambda_{\rm eff} \sim 10^5$ Å, so that $\lambda_{\rm eff}^2 \sim 10^{10}$ Å while $\lambda'\lambda_J \sim 10^{10}$ Å; consequently the Josephson energy is larger. This need not be the case for the Bi and Tl materials, and even more so for the superlattices themselves, for which the Josephson coupling will be much smaller. However, whatever the size of the Josephson coupling, even for a tilted single Abrikosov lattice there must still be essentially the same currents associated with the parallel component of the magnetic field and hence a comparison of these energies is *not* the relevant one.

What *is* relevant is an estimation of the commensurability energy. The interaction between the two Abrikosov lattices tends to make them commensurate. The dominant energy for a weak Josephson coupling is discussed in section 7. The single Abrikosov lattice might be thought of as a special case in the limit that this commensurability energy is very large. Otherwise the picture of the commensurate lattices is a rich one in which some vortex lines tilt while others do not.

When the component of the field in a given direction is less than the appropriate value of $B_{c1}$ for that direction, then the flux lattice *must* tilt.

## 3. The model

In the presence of an *isolated vortex* at the origin and orientated in the $z$ direction (i.e. perpendicular to the plane of the sheets) London's equation for the current density reads (Pearl 1964, 1965, de Gennes 1989),

$$j = \frac{c}{4\pi\lambda^2}(A - \Phi) \tag{1}$$

where $\Phi_\theta = \phi_0/2\pi r$ is the only finite component of $\Phi$, which represents the vortex.

The sheets have a thickness $d$ ($< 10$ Å) and are assumed sufficiently thin so that the current is uniform along the $z$ axis. This is certainly the case if $d < \xi$, where $\xi$ is the coherence length for the material which comprises the sheets. In the context of the high-$T_c$ cuprates, this should be identified with the coherence length ($\sim 30$ Å) associated with the $xy$ plane. The shorter length associated with the $z$ direction ($\sim d$) is small in the present picture because this reflects the weakness of the Josephson coupling between the sheets, rather than the properties of the sheets themselves. However, it is sufficient to have $d < \lambda$, since this is the shortest length over which the current can change appreciably; this condition is satisfied in the experiments on superlattices of which the author is aware. Given that the current is uniform in the $z$ direction, it is natural to define the current per unit length on the sheet,

$$J = dj \tag{2}$$

whence,

$$J = \frac{c}{4\pi}\frac{1}{\lambda_{\text{eff}}}(\Phi - A) \tag{3}$$

where the effective penetration, or *screening length*, for a single sheet is

$$\lambda_{\text{eff}} = \lambda^2/d. \tag{4}$$

For a single sheet,

$$\text{curl}\,\text{curl}\;A = \text{curl}\,H = \frac{4\pi}{c}j = \frac{1}{\lambda_{\text{eff}}}\delta(z)(\Phi - A) \tag{5}$$

where $\delta(z)$ localizes the current in the film, which is now assumed to be indefinitely thin. The real thickness is easily accounted for as needed. The generalization for a superstack (i.e. for repeated superconducting sheets) is

$$-\nabla^2 A + \frac{1}{\lambda_{\text{eff}}}\sum_m \delta(z - m\ell)A = \frac{1}{\lambda_{\text{eff}}}\sum_m \delta(z - m\ell)\Phi \tag{6}$$

using $\text{curl}\,\text{curl}\;A = -\nabla^2 A$, valid in the London gauge. Equation (6) constitutes the model, in the absence of Josephson coupling.

## 4. Calculation

Adapting de Gennes (1989), the first step is to introduce a Fourier transform for the $xy$ plane and a Fourier series for the $z$ direction; the components are,

$$A_{qn} = \int_0^\ell \mathrm{d}z \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}y \; A(x,y,z) \mathrm{e}^{\mathrm{i}2\pi n z/\ell} \mathrm{e}^{\mathrm{i}q_x x + \mathrm{i}q_y y} \tag{7}$$

*where $\ell$ is the distance between the sheets of the stack.* A similar transform, with the label $n$ contracted, is defined to be,

$$A_q = \frac{1}{\ell} \sum_n A_{qn} . \tag{8}$$

The transform of the periodic source term

$$\Phi \sum_m \delta(z - m\ell) \tag{9}$$

is

$$\Phi_q = \mathrm{i}(\phi_0/q^2)\hat{k} \times q \tag{10}$$

where $\hat{k}$ is the unit vector in the $z$ direction.

With these definitions, the transform of (6) is,

$$\left(q^2 + (2\pi/\ell)^2 n^2\right) A_{qn} + (1/\lambda_{\mathrm{eff}}) \left(A_q - \Phi_q\right) = 0 \tag{11}$$

which is sufficient to determine the Fourier components. Solving gives

$$A_q = -\left(\frac{1}{\ell\lambda_{\mathrm{eff}}} \sum_n \frac{1}{\left(q^2 + \left(\frac{2\pi}{\ell}\right)^2 n^2\right)}\right) \left(A_q - \Phi_q\right) . \tag{12}$$

This contains a sum of the form,

$$S = \sum_{k=0}^{\infty} \frac{1}{k^2(\pi/s)^2 + q^2} = \frac{1}{2q^2} + \frac{s}{2q}\left(\frac{\mathrm{e}^{qs} + \mathrm{e}^{-qs}}{\mathrm{e}^{qs} - \mathrm{e}^{-qs}}\right) . \tag{13}$$

It might be noted that for $s \to 0$ only the first term in the sum contributes (i.e. $S = (1/q^2)$). Physically this limit amounts to an approximation in which *all* variations in the $z$ direction are negligible. Using the full result (13) for the sum $S$, it is found that

$$J_q = \frac{c}{4\pi\lambda_{\mathrm{eff}}} \frac{2q\lambda_{\mathrm{eff}}}{\coth \frac{q\ell}{2} + 2q\lambda_{\mathrm{eff}}} \Phi_q . \tag{14}$$

This is one of the principal results for the superstack. Josephson coupling between the sheets has been ignored since it is not relevant when the field is perpendicular to the sheets.

The analysis of this expression depends upon the size of $\ell$ relative to $\lambda_{\rm eff}$. However, whatever the magnitude of these two quantities, small distances are defined as,

$$r < \ell$$

which implies $q\ell > 1$, whence $\coth \to 1$ and

$$J_q = \frac{c}{4\pi\lambda_{\rm eff}} \frac{2q\lambda_{\rm eff}}{1 + 2q\lambda_{\rm eff}} \Phi_q \tag{15}$$

which is the same result as for a single thin film. If now

$$\ell < \lambda_{\rm eff}$$

then automatically $r < \lambda_{\rm eff}$, i.e. $q\lambda_{\rm eff} > 1$, and the approximation,

$$J_q = \frac{c}{4\pi\lambda_{\rm eff}} \Phi_q \tag{16}$$

is valid. This *is*, indeed, the expression appropriate for a single thin film. However, this is not significant since $r < \lambda_{\rm eff}$ corresponds to a limit for which the thin-film result is the same as for the bulk. This equation implies a current per unit length of

$$J = \frac{c\phi_0}{8\pi^2\lambda_{\rm eff} r} \tag{17}$$

per sheet. However, *unlike the case for a single sheet*, this result *does not* extend out to $r \sim \lambda_{\rm eff}$. Rather, for

$$r > \ell$$

it follows that $\coth \to 2/q\ell$, which leads to

$$J_q = \frac{c\ell}{4\pi\lambda'^2} \frac{(\lambda' q)^2}{1 + (\lambda' q)^2} \Phi_q \tag{18}$$

where the *effective*, and measurable, London penetration depth is

$$\lambda' = \lambda(\ell/d)^{1/2}. \tag{19}$$

Apart from this renormalization of $\lambda$, the result (18) is fully equivalent to that for a bulk type II superconductor. For small distances, such that $r < \lambda'$, the non-screened result (17) still holds. *It follows that for all distances such that* $r < \lambda'$

$$J = \frac{c\phi_0 d}{8\pi^2\lambda^2 r} = \frac{c\phi_0 \ell}{8\pi^2\lambda'^2 r} \tag{20}$$

per unit length per film. Still, for the limit that $\ell < \lambda_{\rm eff}$, but now with

$$r > \lambda'$$

it follows from (18) that,

$$J = \ell \frac{c}{4\pi} \left( \frac{\phi_0}{2\lambda'^2} \right) \frac{e^{-r/\lambda'}}{(2\pi\lambda' r)^{1/2}} \tag{21}$$

*which is again the result for a bulk type II superconductor.* Notice that, given $\ell < \lambda_{\rm eff}$, the relevant length scale is $\lambda'$ and not $\lambda_{\rm eff}$.

The thin-film screening length, $\lambda_{\rm eff}$, only becomes relevant if

$$\ell > \lambda_{\rm eff}.$$

In this case it follows that,

$$J = \frac{c\phi_0 d}{8\pi^2\lambda^2 r} \qquad r < \lambda_{\rm eff} \tag{22}$$

i.e. the same result as (20), but now with a different range of validity. On the other hand when $\ell > r > \lambda_{\rm eff}$ the relevant approximation is to take $q\lambda_{\rm eff}$ to be small so that

$$J_q = \frac{c}{4\pi\lambda_{\rm eff}} \frac{2q\lambda_{\rm eff}}{1 + 2q\lambda_{\rm eff}} \Phi_q \rightarrow \frac{c\phi_0}{2\pi} \frac{ik \times q}{q}. \tag{23}$$

This reduces to

$$J = \frac{c\phi_0}{4\pi^2 r^2} \qquad r > \lambda_{\rm eff} \tag{24}$$

which is the well known form for the distant current in a single sheet. Only when the more extreme inequality

$$r > \ell$$

is satisfied does three-dimensional screening come into play and (21) again becomes relevant.

## 5. The lower critical field $B_{c1}^{\perp}$

Using the results of the last section it is simple to determine $B_{c1}^{\perp}$, i.e. the critical field for the formation of the perpendicular Abrikosov lattice when the applied field is perpendicular to the sheets. In the usual fashion, this is determined by the condition that a single vortex is stable, i.e. that $B_{c1}^{\perp} M = U$, where $U$ is the total energy cost of an isolated vortex, and $M$ is the associated magnetic moment.

For a superstack such that $\ell < \lambda_{\rm eff}$, the bulk result applies and (see Pearl 1965),

$$U \sim d\,(\phi_0/4\pi\lambda')^2 \ln(\lambda'/\xi) \tag{25}$$

while $M = \phi_0 d/4\pi$ for both per sheet. Since both the energy and the moment are associated with currents flowing in a single sheet, the result is

$$B_{c1}^{\perp} \sim (\phi_0/\pi\lambda'^2) \ln(\lambda'/\xi) \tag{26}$$

which is the same as for a bulk type II material with a London penetration depth $\lambda'$. Here $\xi$, the coherence length, is, as usual, the relevant small-distance cut-off.

In contrast, when $\ell > \lambda_{\rm eff}$ the single sheet results *are* relevant. However, unlike for the single sheet, the integral for $M$ does *not* diverge, since for a stack there is a cut-off at $\ell$ when bulk behaviour for $J$ again becomes relevant. The larger part of the integral is,

$$M \sim \frac{1}{c}\int AJ\,{\rm d}r \sim \int_\lambda^\ell {\rm d}r\;\pi r^2 \frac{\phi_0}{4\pi^2 r^2} = \int_\lambda^\ell {\rm d}r\;\pi\frac{\phi_0}{4\pi^2} \sim \pi\frac{\phi_0}{4\pi^2}\ell. \tag{27}$$

For a sheet, the energy for a single vortex is

$$U \sim d\,(\phi_0/4\pi\lambda')^2 \ln(\lambda_{\rm eff}/\xi) \tag{28}$$

again per thin film. The only change in the energy, relative to (25), is due to a change in the large-distance cut-off in the logarithm, which is now $\lambda_{\rm eff}$ rather than simply $\lambda$. The ratio $U/M$ gives the result,

$$B_{\rm c1}^{\perp} \sim (d/\ell)\left(\phi_0/\pi\lambda'^2\right)\ln(\lambda_{\rm eff}/\xi)) \tag{29}$$

i.e. compared to the bulk result, the estimate for $B_{\rm c1}^{\perp}$ is smaller by a factor of roughly $d/\ell$. Since for the cuprate superconductors themselves the spacing between the CuO bi-planes, $\ell$, is much less that $\lambda_{\rm eff}$, it is concluded that the superstack model for this system predicts bulk type II superconductor behaviour when the field is perpendicular to the sheets. However, superlattices made from these materials might be made in which $\ell \gg \lambda_{\rm eff} = \lambda^2/d$, whence $B_{\rm c1}^{\perp}$ is reduced compared to the $\ell$-independent bulk result by a factor of $d/\ell$. *The measurement of $B_{\rm c1}^{\perp}$ versus $\ell$ therefore represents a direct method of measuring $\lambda_{\rm eff}$.*

As usual, provided $B \ll B_{\rm c2}^{\perp}$, the problem of the Abrikosov lattice can be considered in terms of a set of interacting single vortices. Ignoring any anisotropy of the superconductivity in the $xy$ plane, the lattice constant $a$ of the vortex lattice is determined by a relationship of the form

$$a \sim (\phi_0/B)^{1/2} \tag{30}$$

where $B$ is the magnitude of the applied field.

## 6. The lower critical field $B_{\rm c1}^{\|}$

In the absence of Josephson coupling between the sheets of the stack, for $\boldsymbol{B}$ parallel to the sheets, the value of $B_{\rm c1}^{\|}$ is zero.

For any other orientation the perpendicular component will penetrate the sample for all fields. An Abrikosov lattice will form on the $xy$ plane at

$$B_{\rm c1} = B_{\rm c1}^{\perp}/\cos\theta \tag{31}$$

i.e. when $B_z = B\cos\theta$ exceeds the critical field $B_{\rm c1}^{\perp}$. Here $\theta$ is the angle that the field makes with the $z$ axis. Since the parallel field between the sheets of the stack is constant, there can be no dependence of the energy upon orientation of the Abrikosov lattice, which is therefore arbitrary and triangular with a constant $a \sim (\phi_0/B_\perp)^{1/2} = (\phi_0/B\cos\theta)^{1/2}$.

Even weak Josephson coupling between the sheets of the stack modifies this picture. The local *current density*, in the $z$ direction (see, for example, Barone and Paterno 1982) is

$$j_z = j_0 \sin\phi \tag{32}$$

where $\phi$ is the phase difference of the order parameters between the two sheets of the stack of interest, and $j_0$ is the critical current density. The energy *per unit area* corresponding to this is,

$$f = \frac{\hbar}{e} j_0 \left[1 - \cos\phi + \frac{\lambda_J{}^2}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}y}\right)^2\right] \tag{33}$$

where $\lambda_J$ is the *Josephson penetration depth*. That such an expression is valid for the coupling between any two sheets of the stack simply amounts to a parametrization of the relevant free energy.

From the theory of long Josephson junctions the predictions for this form for the free energy are known. A finite parallel field penetrates a distance $\lambda_J$ from the edge and a distance $\sim \lambda'$ into the stack. An estimate for $B_{1c}^{\parallel}$ corresponds to the field for which there is one flux quantum in this area, i.e.,

$$B_{c1}^{\parallel} \sim \phi_0/\lambda'\lambda_J. \tag{34}$$

A more accurate estimate is obtained by comparing the energy $F_1$ for an isolated vortex with magnetic energy $MB_{c1}^{\parallel}$ where the magnetization for an isolated linear flux line $M = \phi_0/4\pi$ per unit length. This energy is

$$F_1 = \frac{\hbar}{e} j_0 \int_{-\infty}^{\infty} \mathrm{d}y \left[1 - \cos\phi + \frac{\lambda_J{}^2}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}y}\right)^2\right] \tag{35}$$

where for an isolated vortex

$$\phi = 2\sin^{-1}\operatorname{sech}(y/\lambda_J). \tag{36}$$

The integral gives

$$F_1 = 4\frac{\hbar}{e} j_0 \int_{-\infty}^{\infty} \mathrm{d}y \operatorname{sech}^2\frac{y}{\lambda_J} = 4\frac{\hbar}{e} j_0 \lambda_J \int_{-\infty}^{\infty} \mathrm{d}y \operatorname{sech}^2 y = 8\frac{\hbar}{e} j_0 \lambda_J \tag{37}$$

whence

$$B_{c1}^{\parallel} = F_1/(\phi_0/4\pi) = (32\pi\hbar/\phi_0 e) j_0 \lambda_J \tag{38}$$

which can be equated to the previous estimate

$$(32\pi\hbar/\phi_0 e) j_0 \lambda_J \sim \phi_0/\lambda'\lambda_J. \tag{39}$$

This gives

$$j_0 \sim \phi_0 c/32\lambda' {\lambda_J}^2 \tag{40}$$

as a useful relationship between the Josephson current density and the measurable quantities $\lambda'$ and $\lambda_J$. Eliminating the lengths in favour of fields gives

$$j_0 \sim (c/16)\left(B_{c1}^{\parallel}\right)^2 (1/\phi_0 \pi B_{c1}^{\perp})^{1/2}. \tag{41}$$

It might be noted that the energy of a free vortex parallel to the sheets is ${\phi_0}^2/4\lambda'\lambda_J$, and it does not have any logarithmic dependence upon the size of the system. It follows that Kosterlitz–Thouless considerations are *not* relevant for vortices with this orientation.

When the applied field is parallel to the sheets then there will be a non-symmetric Abrikosov lattice. However the effect is one of scales. Intrinsic pinning favours the alignment of the rows of the lattice so that they are parallel to the plane of the sheets. The two lattice spacings are

$$a^{\parallel} \sim (\lambda_J/\lambda')^{1/2} (\phi_0/B)^{1/2} \qquad a^{\perp} \sim (\lambda'/\lambda_J)^{1/2} (\phi_0/B)^{1/2} \tag{42}$$

so that $a^{\parallel} a^{\perp} \sim (\phi_0/B)$.

## 7. Interaction between Abrikosov lattices

When both $B \sin\theta > B_{c1}^{\parallel}$ and $B\cos\theta > B_{c1}^{\perp}$, then two Abrikosov vortex lattices can exist; necessarily there are interactions between them which tend to make these lattices commensurate. The conventional picture of an anisotropic superconductor with tilted vortices corresponds to the case when these lattices *are* commensurate and have the same wavelength. This is a special case which will certainly not have the lowest free energy for weakly Josephson coupled sheets.

Of the several possibilities the largest interaction which the author has found is the suppression of the Josephson coupling by the normal core of the vortices in the $z$ direction.

The estimation of this energy goes as follows: the energy per unit area per pair of sheets is, again

$$f = \frac{\hbar}{e} j_0 \left[1 - \cos\phi + \frac{{\lambda_J}^2}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}y}\right)^2\right]. \tag{43}$$

For any modest field $B > B_{c1}^{\parallel}$ the field between the sheets is essentially a constant. The pairs of sheets are of two types; those which have vortices between them and those which have not. If there are no vortices between a pair of sheets then the phase difference $\phi$ between adjacent sheets remains in the range 0 to $2\pi$. For a pair of sheets which contain a line of vortices, to a good approximation (i.e. for $B > B_{c1}^{\parallel}$),

$$\phi = ky \qquad k = (2ed_M/\hbar c)B \tag{44}$$

where $d_M \sim 2\lambda'$ is the magnetic thickness. It follows, that the energy associated with this pair of sheets is

$$f = \frac{\hbar}{e} j_0 \left[1 - \cos ky + \frac{{\lambda_J}^2}{2} k^2\right]. \tag{45}$$

The net energy per sheet per vortex, $U_p$, is the energy gain obtained by suppressing the Josephson current in an area $\sim \pi\xi^2$ in a region where the Josephson coupling energy is a maximum in the positive sense, i.e.,

$$\Delta f = \pi\xi^2 2(\hbar/e) j_0. \tag{46}$$

This gives an energy per vortex per sheet of

$$\Delta f = \frac{\pi^3}{2}\left(\frac{\phi_0^2}{4\pi\lambda}\right)\frac{\xi}{d}\frac{\lambda}{\lambda_J}\frac{\xi}{\lambda_J} \tag{47}$$

where the term in the parentheses is an energy per sheet of a single vortex, which can be identified with the Kosterlitz–Thouless temperature $T^0_{KT}$ for a single sheet (Beasley *et al* 1979). Therefore the commensurability energy per sheet per vortex can be written in the form

$$E^s_p = \frac{\pi^3}{2} k_B T_{KT} \frac{\xi}{d}\frac{\lambda}{\lambda_J}\frac{\xi}{\lambda_J}. \tag{48}$$

With realistic estimates $\lambda_{eff}$ is quite large, at least $\sim 10\,\mu$m, and the present model of superconducting stacks might be expected to have a transition temperature related to $T^0_{KT}$ ,except renormalized by the Josephson interactions between planes ($T^0_{KT} \sim 1$ cm $/\lambda_{eff}$ which is $\sim 1000$ K for $\lambda_{eff} \sim 10\,\mu$m!). Details of these ideas will be presented elsewhere. Here we will make the reasonable assumption that the superconductive transition temperature $T_c \sim T^0_{KT}$. Since it is also assumed that $T \sim T_c$, it is necessary for the pinning energy to be of the order of $T^0_{KT}$ for it to be considered appreciable.

Since all of the ratios in the above, except for $\xi/d$, are small, $E^s_p$ is small compared to $T$ or $T_{KT}$. However, this must be multiplied by factors which arise because of the effective 'coherence lengths' associated with the Abrikosov lattices. These are defined in terms of the distance over which the two lattices remain in synchronization. The relevant lengths are defined to be $L^\perp$ for the $xy$ plane and $L^\parallel$ for the the $z$ direction. The total number of sheets involved is $\sim L^\parallel/a^\perp$ and the total number of vortices is $\sim (L^\perp/a)^2$, so the total commensurability energy is

$$E^t_p = \frac{\pi^3}{2} k_B T_{KT} \frac{\xi}{d}\frac{\lambda}{\lambda_J}\frac{\xi}{\lambda_J}\frac{L^\parallel}{a^\perp}\left(\frac{L^\perp}{a}\right)^2. \tag{49}$$

An estimate of this quantity is very much dependent on the quality of the sample. (It might be noted that the above-defined coherence lengths are *not* the same as those associated with long-range order for either of the Abrikosov lattices. For a reasonable commensurability potential it is to be expected that the lattices remain in synchronization for much greater distances than the distances over which long-range order is maintained.)

Clearly when this interaction is important then the two lattices tend to be commensurate. A full discussion of the possible commensurabilities is lengthy. The simplest case is to have

$$na = ma^{\parallel} \tag{50}$$

where $n$ and $m$ are integers. This expresses the idea that the vortices parallel to the sheets match up with the rows of the triangular lattice. Clearly the effect of these interactions is to orientate the Abrikosov lattice in the $xy$ plane relative to the vortex lines which run parallel to the sheets. The orientation is not correlated to the $a$ and $b$ axes of the underlying lattice (except for the possibility that the pinning reflects this lattice). This is in accord with the experiments of Bolle *et al* (1991).

When this condition is not satisfied, or satisfied only with large numbers, it becomes energetically advantageous to add 'extra' vortices to the 'rows' defined by the parallel lattice in order to maintain commensurability (with low numbers). This is a natural explanation for the 'chains' observed in the experiments of Bolle *et al* (1991).

## 8. Conclusions

It is widely assumed that a stack of Josephson-coupled superconducting planes—a superstack—can be treated as a highly anisotropic superconductor. Whatever the angle the applied field makes with the principal axes, in such a treatment of the mixed state there is a single Abrikosov lattice. In general the vortices are highly anisotropic. It has been shown here that a superstack (i.e. a superlattice made of Josephson-coupled sheets of superconductor) has two Abrikosov lattices when the field is not close to one of the principal axes. While these ideas have been developed in the context of superlattices, it is also envisaged that the superconducting cuprates *themselves*, e.g. $YBa_2Cu_3O_{7-\delta}$, represent the simplest case corresponding to a superstack.

It has been shown that the crossover from two-dimensional, single-sheet behaviour, to three-dimensional bulk type II superconductor properties occurs when the separation between the sheets, $\ell$, is of the order of $\lambda_{eff}$, the screening length for a single sheet (the length, via $k_B T_{KT} \sim \phi_0^2/\lambda_{eff}$, determines the Kosterlitz–Thouless temperature). The observable London penetration depth is $\lambda' = (\ell/d)^{1/2}\lambda$, where $d$ is the thickness of the sheets and $\lambda$ is the intrinsic penetration depth.

Corresponding to the two Abrikosov lattices are two $B_{c1}$ values; one $B_{c1}^{\parallel}$ (perhaps $\sim 100$ G for $YBa_2Cu_3O_{7-\delta}$) for the field parallel to the sheets and another $B_{c1}^{\perp}$ ($\sim$ 1 G) for the perpendicular direction. For a field in an arbitrary direction there remain two values of $B_{c1}$ associated with the establishment of the two Abrikosov lattices, e.g. if the parallel component of $B$ exceeds $B_{c1}^{\parallel}$ then the parallel Abrikosov lattice forms; the perpendicular lattice forms independently only when the perpendicular component exceeds $B_{c1}^{\perp}$. For weak Josephson coupling $B_{c1}^{\parallel} \ll B_{c1}^{\perp}$, as observed experimentally.

There *is* an interaction between the two Abrikosov lattices and this tends to favour commensurability. Close to commensurability, or when this can only be attained in high order, it becomes advantageous for extra flux to be accommodated by the formation of chains on the surface of the sheets with the chains coinciding with the centre of vortices parallel to the sheets.

It would appear that this description is compatible with recent vortex decoration experiments of Bolle *et al* (1991).

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